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A Tale of Tails: Uncertainty and the Social Cost of Carbon Dioxide

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Abstract Recent thinking about the economics of climate change has concerned the uncertainty about the upper bound of both climate sensitivity to greenhouse gases and the damages that might occur at high temperatures. This argument suggests that the appropriate probability distributions for these factors may be fat-tailed. The matter of tail shape has important implications for the calculation of the social cost of carbon dioxide ($SCCO_2$). In this paper a probabilistic integrated assessment model is adapted to allow for the possibility of a thin, intermediate or fat tail for both (i) the climate sensitivity parameter and (ii) the damage function exponent. Results show that depending on the tail shape of the climate sensitivity parameter the mean $SCCO_2$ rises by 29–85%. Changes in the mean $SCCO_2$ due to the adjustments to the damage function alone range from a reduction of 7% to a rise of 12%. The combination of both leads to rises of 33–15%. Greater rises occur for the upper percentiles of the $SCCO_2$ estimates. Given the uncertainties in both the science and the economics of climate change different tail shapes deserve consideration due to their important implications for the range of possible values for the $SCCO_2$.

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Keywords Climate change; integrated assessment models; social cost of carbon dioxide; uncertainty.

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1 Introduction

In the study of the economics of climate change, the issue of how to deal with catastrophic events has recently received a great deal of attention. The possibility that climate change could cause catastrophic outcomes is of deep concern, even if such an outcome is unlikely. From a scientific perspective, it has long been clear that radically changing the composition of the atmosphere, effectively instantaneously in geological terms, could have large, irreversible effects on ecosystems and highly undesirable consequences for humankind. This leads to calls for urgent action to limit the concentrations of greenhouse gases (GHGs) (IPCC 2007; European Commission 2007) or even to reduce concentrations below their current levels (Hansen et al. 2008).

There is some controversy within the economics literature regarding how to deal with climate impacts and what the policy implications are (e.g. Weitzman 2009b; Nordhaus 2009). On the one hand, many of the economic assessments of the damage from carbon emissions report fairly modest figures (for an overview, see Tol 2008; for a recent estimate, see Interagency Working Group on Social Cost of Carbon, U.S. Government 2010). When these estimates are placed into a cost-benefit analysis framework, which balances *inter alia* medium-term prosperity against longer-term damages, the resulting recommendations can be for GHG concentrations to rise above the limits recommended by the IPCC.[1] On the other hand, a more recent literature (especially Weitzman 2007 and 2009b) has suggested that the possibility of catastrophic events could be a key factor (perhaps *the* key factor) in climate economics, even if such events are unlikely.

It is not the intention of this article to make a comprehensive review of this discussion. Instead, the investigation focuses on the treatment of catastrophic events within the economic estimates of the damage from GHGs. Though some previous economic estimates have allowed for some possibility of catastrophic damage, recent research (Weitzman 2010; Dietz 2011a) proposes further directions in which the relevant models can be adapted to better account for this factor.

Climate damages are typically estimated with stylised integrated assessment models (IAMs), which take into account contributions to climate policy from vari-

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[1] This criticism is made in Weitzman (2010), with reference to results from Nordhaus' DICE model.

ous disciplines, from climatology to economics. These model the most significant interactions and feedback mechanisms of the human-climate system. They also deal with intergenerational fairness, income regional distribution and, some of them, at least to a certain extent, risk and uncertainty management (Dietz et al. 2007).

A typical application of IAMs is the computation of the social cost of carbon dioxide ($SCCO_2$), i.e., the cost to society caused by one additional tonne of carbon dioxide released into the atmosphere. The $SCCO_2$ is a prominent indicator within both the literature and the policy debate. In principle, it summarizes climate policy benefits in a single dimension variable taking into account all possible biophysical and economic impacts in all world regions, all future time periods and in all future contingencies. The concept is particularly useful in project appraisal putting a value to the benefits of avoided GHGs emissions.[2] The value of the $SCCO_2$ can be useful in judging policies, which in economic terms are only justified when their marginal benefit is at least equal to their marginal cost. It can also be used to guide the setting of a cap in a cap-and-trade scheme.

Inevitably, IAMs rely on a series of simplifying assumptions,[3] using highly aggregated variables and data (Ackerman et al. 2009a; Patt et al. 2010), and the limitations of the methodology have been noted (e.g. Warren et al. 2006, Dietz et al. 2007). Nevertheless, IAMs provide a useful conceptual framework for exploring the implications of alternative specifications. Furthermore, many IAMs are designed to be flexible Nordhaus and Boyer 2000, allowing users the opportunity to enter alternative parameters. As noted in Dietz et al. (2007), IAMs can be thought of as a "canvas" on which debates about the parameters can be "painted". We take advantage of this feature in this research.

This article moves from the discussion in Weitzman (2010), questioning the extent to which uncertain extreme values of climate sensitivity and the damage functions have been accounted for in IAMs. In particular it explores a method of introducing thin, intermediate and fat tails for these key parameters in a specific IAM, the PAGE09 Model. With respect to the climate sensitivity, the alternative

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[2] For a comprehensive discussion of the $SCCO_2$, including its main weaknesses, see for instance Ackerman and Stanton (2010).

[3] DICE 1990 model is based for instance on twenty equations (Nordhaus and Boyer 2000).

tails proposed in that paper are inputted and the damage functions are extended in the spirit of Weitzman's analysis. The focus is specifically on the implications for these changes on the $SCCO_2$.

The article is organised as follows: Section 2 is devoted to methodological issues and explains the model used and the changes introduced on both the sensitivity parameter and the damage function exponents; Section 3 presents the main results and Section 4 concludes.

2 Methodology

2.1 The PAGE09 Model

The latest version of the PAGE model, PAGE09, keeps unchanged the general structure of the version used for the Stern Review (Stern 2007), but introduces further developments reflecting the IPCC Fourth Assessment Report (2007). Exogenous assumptions for economic and population growth and GHGs emissions reflect the IPCC SRES A1B scenario (Hope 2011).

PAGE09 uses a simple economic module (Hope et al. 1993; Plambeck et al. 1997; Hope 2006; Hope 2008; Hope 2011) and expands it to consider climate issues and the linkages between the economic and the climate systems through some stylized equations within the climate module. Uncertainty is taken into account through Latin Hypercube sampling[4]. Functional forms are assumed to be known with certainty, while each of the uncertain model parameters (approximately 80) is represented by a probability distribution. The discount rate in our model is unaltered from the PAGE09 standard form, which discounts according to an equity weighting scheme, and then by the rate of pure time preference Hope 2011. The equity weighting scheme converts changes in consumption into utility giving more (less) weight to consumption per capita in poorer (richer) regions and time periods. The weighting is dependent on the elasticity of marginal utility of

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[4] Latin Hypercube sampling is preferred to "random" Monte Carlo sampling since it provides a better coverage of the underlying PDFs.

consumption, which is entered as a triangle distribution (0.5, 1, 2).[5] Following the equity weighting, the damages are further discounted at the rate of pure time preference, which is entered as a triangle distribution (0.1, 1, 2) measured in percentage per year. A full run of the model involves repeating the calculations of the following output variables: global warming over time, damages, adaptive costs and abatement costs.

Four impact categories, specified as the percentage loss of GDP and subtracted from consumption, are defined within the economic module: sea level impact, economic and non-economic impacts based on regional temperature rise and discontinuity impact. As in most IAMs, damage is defined as a non-linear function (Bosello and Roson 2007). The total effect of climate change is equal to the sum of impacts, abatement costs and adaptive costs.[6]

2.2 Uncertainty in the climate sensitivity parameter (SENS)

2.2.1 Background and Literature

Weitzman (2010) provides a methodology to model highly uncertain economic consequences induced by catastrophic climate change events. Due to their unknown and potentially huge consequences for humankind, even low probability events associated with highly-negative impacts need to be taken into account in the economics of climate change. With respect to temperature changes, Meehl et al.(2007)[7] suggest that the likelihood of very high temperature rises is greater than current IAMs allow for. Generally, both the temperature response to GHGs (Weitzman (2009a)) and the consequent economic damages are highly uncertain.

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[5] The formula used amounts to weighting consumption as the following: $E(r,t) = [GDP_i(EU,0)/GDP_i(r,t)]^{EMUC}$, where $E(r,t)$ is the equity weighting by region and time period, $GDP_i(EU,0)$ is the gross domestic product per capita in the EU in the initial period and $GDP_i(r,t)$ is the gross domestic product per capita by region and time period, and $EMUC$ is the elasticity of marginal utility of consumption. The GDP_i values can be derived from the initial values and growth rates for gross domestic product and population given in Hope (2011).

[6] Since in standard welfare models with constant and strictly positive relative risk aversion marginal utility tends to infinity as consumption tends to zero, if climate damages can reach 100% of consumption, then they need to be in some way bounded (Dietz 2011a). Following a suggestion in Weitzman (2009b), total damages are capped if they exceed the statistical value of civilisation.

[7] This study provides 22 estimates of climate sensitivity.

While clearly, it is impossible to know the "true" probability distribution, the general notion that there are small – but decidedly nonzero – probabilities of extreme events is certainly one that can be incorporated in a modelling framework. This is a belief that can be "painted" onto the "canvas" of an IAM.

The intention is to take account of the uncertainty surrounding both the physical processes governing temperatures and the economic valuation of the welfare losses associated with catastrophic events. The first type of uncertainty might be captured by the so-called equilibrium climate sensitivity (SENS) parameter. As stated in Meehl et al.(2007), this provides "a measure of the climate system response to sustained radiative forcing" and is defined as "the global average surface warming induced by a doubling of carbon dioxide atmospheric concentration after a new equilibrium of the climate system has been reached. It is likely to be in the range 2 to 4.5°C with a best estimate of 3°C and is very unlikely to be less than 1.5°C. Values substantially higher than 4.5°C cannot be excluded, but agreement of models with observations is not as good for those values".

With respect to this research, it is more accurate to interpret climate sensitivity as a summary for the consequences of climate change, many of which are highly uncertain.[8] Focusing on climate sensitivity is, therefore, a reductionist approach. With respect to the science, it can be justified because, as well as the importance of climate sensitivity in itself, it is also correlated with many aspects of climate change effects (Knutti and Hegerl 2008). This also justifies the prominent role it plays in IAMs, such as PAGE09.

Due to their uncertain nature, temperature changes induced by GHGs atmospheric concentration can only be described in terms of probabilities. In identifying the climate sensitivity probability distribution function (PDF), Weitzman refers to the language of tail probabilities. While the existing literature on cost-benefit analysis and IAMs of climate change mainly focuses on super thin tailed point mass PDFs, he takes into account tails of varying degrees of fatness: thin tailed probabilities, declining exponentially or faster; fat-tailed probabilities, declining polynomially or slower; intermediate-tailed probabilities, declining slower than exponentially but faster than polynomially. As will be shown below, for the up-

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[8] The same argument was made in Weitzman (2010) with respect to the use of climate sensitivity in that paper.

per 50 percentiles his proposed PDFs are implemented: 1) the thin-tailed normal distribution; 2) the fat-tailed Pareto distribution; and 3) the intermediate-tailed lognormal distribution. All three are calibrated so as to have a median of 3°C, which is the best estimate from the IPCC-AR4 (2007). The probability of the value being between 2°C and 4.5°C is reported to be above 66% and below 90%, which leads Weitzman (2010) to propose an 85th percentile of 4.5°C.[9]

2.2.2 Model adjustments

In PAGE09, the SENS parameter is determined by two input variables: transient climate response (TCR) and the feedback response time (FRT). The former refers to the temperature change (°C) at the time of CO_2 concentration doubling. The latter indicates how many years GHGs persist in the atmosphere. The relationship among the three variables is indicated in Equation (1):[10]

$$SENS = \frac{TCR}{1 - \left[\left(\frac{FRT}{70} \right) * \left(1 - e^{\left(\frac{-70}{FRT} \right)} \right) \right]} \tag{1}$$

In other words, the probabilistic distributions of TCR and FRT affect the variable capturing the global temperature increase due to a doubling of CO_2 concentration in the atmosphere. In PAGE09, these variables are assumed to follow a triangular probability distribution. In the modified version of PAGE09, the original distribution of the considered variables is kept, but a different definition of SENS has been introduced, in order to take into account Weitzman's suggestions. In doing so, the SENS distribution has been modified in such a way that up to its

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[9] The IPCC likely probability definition implies a probability between 5 and 17% of climate sensitivity being greater than 4.5°C. Weitzman justifies edging towards the high end of that range as the "earth system sensitivity" probably matters more than the "fast equilibrium sensitivity" over the relevant time frame. Zickfield et al. (2010) estimate the same probability to be equal to 23%, while in Pindyck (2011) it is equal to 10%.

[10] While in PAGE2002 SENS was inputted directly as an uncertain parameter, its calculation discussed here is an innovation of PAGE09 based on IPCC 4AR and research by Andrews and Allen (2008). These authors introduced the feedback response time (FRT) in order to correct for the undersampling of the range of transient climate responses (TCR) consistent with recent attributable greenhouse warming.

median it is distributed according to a rescaled version of the original PDF, while for its upper-half tail assumes the distributions discussed by Weitzman (2010). The PDFs used are explained in detail below. Further discussion about why these PDFs were chosen in preference to other alternatives is given in Appendix A.

(a) Lower 50 percentiles

For the lower 50 percentiles, the standard calibration is kept and therefore the standard PAGE09 values are retained: TCR sampled between 1 and 2.8°C – triangle (1, 1.3, 2.8) – and FRT between 10 and 65 years – triangle (10, 30, 65). The values below 3°C are drawn proportionally for the lower 50 percentiles. In the standard version of the model, 55.62%[11] of the results are less than 3°C. Therefore, to obtain 50% of the final draws, this distribution is sampled whenever the value is (i) below 3°C and (ii) a uniform distribution [0, 1] is below 0.8990.[12]

(b) Upper 50 percentiles

For the upper 50 percentiles of SENS, the thin, intermediate and fat-tailed distributions proposed in Weitzman (2010) are inputted, with the corresponding parameter values. As he suggests, the median is fixed at 3°C, which is consistent with the best estimate of the IPCC-4AR (2007), and the 85th percentile at 4.5°C. This comes from the "likely" range given by the IPCC of 2 to 4.5°C, where "likely" is defined as a probability greater than 66% but less than 90%. Defining "likely" as 70%, gives the 85th percentile of 4.5°C. Given these two parameter values, the three distributions proposed by Weitzman, the thin-tailed normal distribution, the fat-tailed Pareto distribution and the intermediate-tailed lognormal distribution, are taken

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[11] Based on one million runs.

[12] 55.62% * 89.90% = 50.00%. Note that this procedure slightly raises the median value of SENS relative to the standard model, whose median is equal to 2.87°C (based on 10,000 runs).

into account.[13] Fitting these distributions to the specified 50th and 85th percentiles gives the following values:

$$\text{Normal: } f_N(SENS) = \frac{1}{1.447\sqrt{2\pi}} e^{\left(-\frac{(SENS-3)^2}{2\times(1.447)^2}\right)} \tag{2}$$

$$\text{Lognormal: } f_L(SENS) = \frac{1}{0.3912\sqrt{2\pi}SENS} e^{\left(-\frac{(lnSENS-1.099)^2}{2\times(0.3912)^2}\right)} \tag{3}$$

$$\text{Pareto: } f_P(SENS) = 38.76 \times SENS^{-3.969} \tag{4}$$

The three distributions are compared graphically in Figure1.

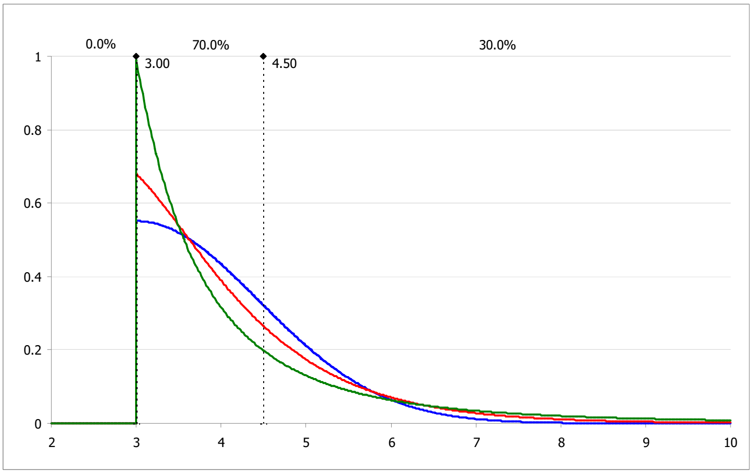

Figure 1: The upper 50 percentiles of the climate sensitivity parameter: normal, lognormal and Pareto

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[13] In his empirical analysis of risk in the economics of climate change, Dietz (2011b) takes into account the previous version of the model (PAGE2002) and considers a log-logistic distribution for the climate sensitivity parameter and a lognormal distribution for the damage function. These two are the distributions best fitting in terms of the lowest root-mean-square error.

The graphs show the differences in tail thickness, with the Pareto (green line) being the fattest tail, followed by the lognormal (red line) and the normal (blue line). Note that both the lognormal and Pareto draw more often from the lower range of values (close to 3) than the normal does.

2.3 Uncertainty in the damage functions

2.3.1 Background and Literature

There is considerable uncertainty about the correct shape of damage functions. The argument is similar to that made for the climate sensitivity parameter above: whilst reasonable estimates can be made for the lower end of the distribution, the high end of the distribution is uncertain, and possibly unknowable.

The damage functions relate to the economic consequences caused by the physical response of the climate system. When attempting to quantify climate change damages, one is trying to estimate the net cost of damage from sources such as population movements, damage to property, agricultural productivity, access to fresh water and, generally, access to what can be termed bio-system services. Clearly, there is some difficulty in estimating such costs for small temperature changes. As estimates are made for higher temperatures, the difficulties are compounded further. It is uncertain whether the damages for large temperature changes are simply an extrapolation of damages for small temperature changes. Various tipping points can be envisaged (Lenton et al. 2008; Kriegler et al. 2009), which would lead to severe sudden damages. Furthermore, the consequent political or community responses could be even more serious.

Rapid climate change will stress many economic, social and political systems. Of course, it is impossible to predict the result of such events, especially the extreme negative tail, which is why the possibility of very high damages ought to be included in the analysis. The opposite viewpoint–insisting that dramatic consequences will not occur–seems more difficult to justify.

2.3.2 Model adjustments

At the core of the damage function in PAGE09 is the Equation (5).[14]

$$d = \alpha \left(\frac{T_{ACT}}{T_{CAL}} \right)^{\beta} \tag{5}$$

where d is the damage, α is the damage at the calibration temperature, T_{CAL} is the calibration temperature rise, and T_{ACT} is the actual temperature rise, β is the damage exponent.

The calibration temperature is on average 3°C.[15] Therefore, if the actual temperature rise is 3 °C, on average, the damage equals α. The damage exponent, β, becomes more important as temperatures rise above T_{CAL}. In the standard model, β is entered as triangle (1.5, 2, 3). Therefore, on average, the exponent is 2.167 (slightly above a quadratic), meaning that at twice the calibration temperature (on average, T_{ACT} equals 6°C), the damage will be 4.5 times α. With the maximum value for β, which is 3, the damage would be 8 times α.

This shows that the standard PAGE09 Model does allow for the possibility of reasonably high damages. However, the arguments above suggest that these bounds may not adequately take into account the possibility of extreme damages. In the same spirit as for the changes in SENS, three distributions are proposed for the damage exponent, β.

(a) Lower 50 percentiles

The median value for β is chosen to be 2 (i.e. quadratic), which is the most common value for β in the literature.[16] The lower 50 percentiles are entered as the standard model distribution for values below 2. This is simply triangle (1.5, 2, 2).

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[14] The full estimates of damages in PAGE09 take into account regional differentials, discounting, weighting for income inequality, saturation of damage effects and the capacity for adaptation. All such considerations are unchanged from the standard model.

[15] It is entered as a triangle distribution (2.5, 3, 3.5).

[16] Note that the median in the standard PAGE09 model is slightly higher at 2.134.

(b) Upper 50 percentiles

For the upper percentiles, we follow a suggestion of Dietz (2011a), who proposes incorporating a 10% probability that the β exceeds 3. Otherwise, the distributions are fitted to have a median of 2. As for the SENS parameter, three distributions are fitted to these criteria: a thin-tailed normal, an intermediate-tailed lognormal, and a fat-tailed Pareto. Fitting these distributions as specified gives the following PDFs:

$$\text{Normal: } f_N(\beta) = \frac{1}{0.7803\sqrt{2\pi}} e^{\left(-\frac{(\beta-2)^2}{2\times(0.7803)^2}\right)} \tag{6}$$

$$\text{Lognormal: } f_L(\beta) = \frac{1}{0.3164\sqrt{2\pi}\beta} e^{\left(-\frac{(lnSENS-0.6931)^2}{2\times(0.3164)^2}\right)} \tag{7}$$

$$\text{Pareto: } f_P(\beta) = 31.09 \times \beta^{-4.969} \tag{8}$$

The tails of the distributions are shown in Figure 2, which compares the normal distribution (blue line), the lognormal (red line) and the Pareto (green line).

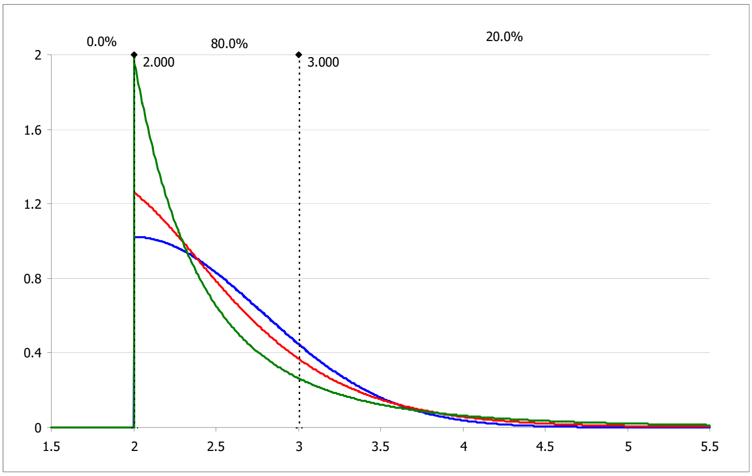

Figure 2: The upper 50 percentiles of the damage exponent: normal, lognormal and Pareto

These distributions were entered for both the economic and non-economic damage functions. The third type of damage in PAGE09 comes from sea-level rise. The exponent for sea-level damages in the standard model is triangle (0.5, 0.7, 1). Analogous reasoning to that for economic and non-economic damage exponent is used to adjust the sea-level damage exponent. The resulting distributions all have a median of 0.7 and a 90th percentile of 1. The lower 50 percentiles are triangle distributions (0.5, 0.7, 0.7) and the upper 50 percentiles are normal, lognormal or Pareto.

The fourth damage category in PAGE09 is discontinuity damages, which is intended to account for uncertain damages not elsewhere accounted for in the model. As the justification for discontinuity damages partially overlaps with the justification for extending the tails on the damage exponents, we have switched them off here to be sure to avoid double-counting.[17]

3 Results

The results show significant changes in the estimated probability distribution of the $SCCO_2$ when different tails are inputted for the relevant PDFs. Following the steps of our methodology, the results are presented as follows: (i) the effects of changing the climate sensitivity parameter alone, (ii) the effects of changing the damage functions alone, and (iii) both effects together. For reference, the full set of all results is provided in a single table in Appendix B.

3.1 *Climate sensitivity parameter (SENS)*

Table 1 shows the results of changing the probability distribution of the climate sensitivity parameter (SENS) only. The damage functions are unchanged, including keeping the discontinuity damages ("Disc. ON" in Table 1). For comparison, the first column refers to the standard PAGE09 Model, with the default assumptions.

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[17] The same is done in Dietz (2011a).

The standard model provides a mean value of 102 $/tCO$_2$, which is already higher than many of the estimates in the literature.[18]

Table 1: Alternative SENS - SCCO$_2$ in US$/tCO$_2$

	Standard (triangle)	Thin tail (normal)	Interm. tail (logn.)	Fat tail (Pareto)
	Disc. ON	Disc. ON	Disc. ON	Disc. ON
Mean	**102**	**131**	**146**	**188**
5th perc.	11	12	11	12
50th perc.	49	57	57	54
95th perc.	231	374	409	564
99th perc.	447	841	1,095	2,797

Source: Authors' calculations, each based on 10,000 runs of the modified PAGE09.

When the tail for climate sensitivity is normally distributed the SCCO$_2$ mean value rises to 131 $/tCO$_2$ (29% above the standard model). In the case of lognormal distribution for SENS the mean value is 146 $/tCO$_2$ (44% above), while when a Pareto distribution is used the SCCO$_2$is estimated to be 188 $/tCO$_2$ on average (85% above).

It is worth emphasising the asymmetry of the effects on the SCCO$_2$ range. As expected, the values of the 5th and 50th percentiles do not change significantly, while there is a large increase for the 95th and 99th percentiles. Using a normal distribution for climate sensitivity implies that SCCO$_2$ would be larger than 374 $/tCO$_2$ with a 5% probability, and there would be a probability of 1% that the value of SCCO$_2$ is above 841 $/tCO$_2$ (which is 88% higher than the 99th percentile of the standard PAGE09 Model). In the case of the Pareto distribution the value of the 95th percentile is 564 $/tCO$_2$ and that of the 99th percentile[19] is 2,797 $/tCO$_2$,

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[18] Many of the reasons for this (especially the differences between PAGE2002 and PAGE09) are explained in Ackerman et al. (2009b) and Hope (2011).

[19] The 99thpercentile is rarely reported for the estimates of the SCCO$_2$ in the literature. We include it here because in the context of the discussion about climate uncertainty, we believe it is important to acknowledge the extreme values, even if they are highly unlikely. Roughly speaking extended tails for the PDF of the inputs to the model leads to extended tails in the outputs. This is a relevant result,

which corresponds to an increase of 144% and 525% respectively, with respect to the estimates of the standard PAGE09 Model.

3.2 *Damage exponents*

Table 2 reports the results of the runs when the distribution of the damage exponents is modified, without changing the distribution of the sensitivity parameter. In these cases, as discussed in the previous section, the probability of discontinuity damages is switched off, to avoid double counting. In order to distinguish which part of the changes is due to removing the discontinuity damages and which part is due to adding the tails, the standard PAGE09 Model is run with the discontinuity damages switched off (second column of Table 2). This alone gives a significantly lower mean value for the $SCCO_2$, equal to 76 $/$tCO_2$.

Table 2: Alternative damage exponents - $SCCO_2$ in US$/$tCO_2$

	Standard (triangle)	Standard (triangle)	Thin tail (normal)	Interm. tail (logn.)	Fat tail (Pareto)
	Disc. ON	Disc. OFF	Disc. OFF	Disc. OFF	Disc. OFF
Mean	**102**	**76**	**99**	**94**	**114**
5^{th}perc.	11	11	11	11	11
50^{th} perc.	49	48	50	50	50
95^{th} perc.	231	226	300	300	358
99^{th} perc.	447	418	658	762	1,421

Source: Authors' calculations, each based on 10,000 runs of the modified PAGE09.

Modifying the PDFs of the damage exponents raises the $SCCO_2$ (in a similar way as for changing climate sensitivity) relative to the standard model without discontinuity damages. When comparing to the full standard model (with discontinuity damages), adding thin, intermediate or fat tails either lowers the mean value

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which we wish to show. Nevertheless, it should also be acknowledged that, as one would expect, the values for the 99^{th}percentiles relatively imprecise, varying fairly considerably if the same scenario is rerun.

of the $SCCO_2$ by 3 or 7% or raises it by 12% respectively (99,94 and 114 \$/tCO_2 compared to 102).

3.3 *SENS and damage exponents together*

Table 3 shows the results of the model when both changes (on SENS and the damage exponents) are done in combination.

Table 3: Alternative SENS and damage exponents - $SCCO_2$ in US\$/tCO_2

	Standard (triangle)	Thin tail (normal)	Interm. tail (logn.)	Fat tail (Pareto)
	Disc. ON	Disc. OFF	Disc. OFF	Disc. OFF
Mean	**102**	**135**	**147**	**218**
5th perc.	11	12	12	11
50th perc.	49	58	57	55
95th perc.	231	489	551	839
99th perc.	447	1,276	1,660	3,082

Source: Authors' calculations, each based on 10,000 runs of the modified PAGE09.

The adjusted model, with both SENS and the damage exponents normally distributed, estimates the mean $SCCO_2$ to be 135 \$/tCO_2on average, 147 \$/tCO_2 with the lognormal distribution and 216 \$/tCO_2 with the Pareto distribution. The new $SCCO_2$ is 33 to 115% higher than the standard PAGE09 Model.

As in the previous cases, the lower half of the distribution is essentially unchanged (adjusting the tail has little impact on the bulk of the results). The upper percentiles, however, are greatly extended. The 95th percentile shows rises from between 110 to 260%, while the 99th percentile shows even greater rises.[20]

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[20] The reason for this behaviour of the model is partly a consequence of the way extreme damages are modelled. In the standard PAGE09 model, the discontinuity damages are large if they occur. The largest values for the $SCCO_2$ are usually recorded in the runs where the marginal increase in CO_2 causes such a discontinuity, which would not otherwise have occurred. However, this is rare, occurring in less than 1% of the runs. Therefore such values are not included in even the 99th percentile in the standard PAGE09 results. As noted above, the discontinuity damage feature has

3.4 *Comparison with existing estimates*

Many estimates of the $SCCO_2$[21] have been proposed in the literature. Some early estimates include Fankhauser (1994) who reports marginal impacts of between 2 and 12 \$/tCO_2 with a mean value of 5 \$/tCO_2 (figures in US\$1990). The Second Assessment Report from the IPCC (1996) estimates range from 1 to 34 \$/tCO_2 (US\$1990). Tol (1999) estimates the marginal impact to be between 2 and 6 \$/tCO_2 (US\$1990). Tol (2005) gathered over 100 estimates from 28 published studies and combined them to form a probability density function with a median of 4 \$/tCO_2, a mean of 25 \$/tCO_2, and a 95th percentile of 95 \$/tCO_2. In an updated version of this meta-analysis, Tol (2008) considered 211 estimates of the SCC and found higher estimates than in the previous studies. Adjusting alternative kernel density estimators to data points, the author found that when the Gaussian distribution and the sample coefficient of variation is used (which is the case closest to the 2005 study), the distribution of the estimates has a median of 4 \$/tCO_2, a mean of 28 \$/tCO_2 and a 95th(99th) percentile of 162 \$/tCO_2 (552 \$/tCO_2). The Stern Review Stern 2007, which uses the PAGE2002 model, estimates a $SCCO_2$ of 85 \$/tCO_2 (US\$ 2000).

A recent report of the US Government Interagency Working Group on Social Cost of Carbon (2010) presents $SCCO_2$ estimates resulting from three IAMs, the DICE, PAGE2002 and FUND models. The $SCCO_2$ estimates from the average

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been switched off for the new simulations, as the justification for it overlaps with the justification for adding the tails probabilities.

Investigations into $SCCO_2$ estimates could be done by adjusting the probabilities concerning discontinuity damages (tolerable temperature rise without discontinuity, chance of discontinuity, loss if discontinuity occurs) allowing, for example, a higher chance of discontinuity. However, the discontinuity impact is a highly aggregated measure, incorporating risks and uncertainties of several different impacts, which cannot be identified individually. Therefore we suggest that the fat-tails approach in individual physical and economic damages is a more transparent way to include extreme events in the model, reflecting the structure of current scientific information about climate risks.

[21] Note that many of the results quoted here were originally reported as estimates of the social cost of carbon (not of carbon dioxide). These have been converted into $SCCO_2$ units. The multiplier for doing so is the relative molecular weight of CO_2 to carbon, which is 3.67 (44 g per mole/12 g per mole;Interagency Working Group on Social Cost of Carbon, U.S. Government (2010)). For example, this means that 100 \$/tCO_2 is equivalent to 367 \$/tC.

of the three IAMs are 35,21 and 5 $/tCO$_2$, at discount rates of 2.5%, 3% and 5%, respectively.

Ackerman and Stanton (2011) make three key adjustments to the DICE model. The paper adjusts the damage function based on Hanemann's work for low temperatures and Weitzman's for high temperatures. A lower discount rate is also experimented with. All these changes together suggest a mean SCCO$_2$ of 481 $/tCO$_2$ and a 95thpercentile of 893 $/tCO$_2$ in 2010.

With the exception of Ackerman and Stanton (2011), our results appear to be significantly higher than the average SCCO$_2$ estimates provided in the literature so far. Nevertheless, comparing our results with past estimates is not straightforward, as differences in the model structure and parameter values,[22] emissions and socioeconomic scenarios and discount factor assumptions might bias the effects of introducing uncertainties in key parameters. While we can directly compare the results of the PAGE09 model before and after fat-tail adjustments, further research would be needed to isolate the impact of these adjustments with respect to the different models and model assumptions used in previous studies.

4 Conclusions

There are large uncertainties surrounding the catastrophic impacts climate change might cause. These uncertainties call into question the appropriate weight in the tails of the probability distributions in climate models. Following recent literature and taking advantage of the flexible nature of IAMs, we have adjusted the tails of two key areas of uncertainty in the PAGE09 Model: the climate sensitivity parameter and the damage exponents. For each, we considered a normal (thin), a lognormal (intermediate) and a Pareto (fat) tail. Though there are some doubts about the probabilities for the bulk of these distributions, we focused on the extreme values which are the most uncertain.

Which of the three tail shapes is the most credible can be debated. Under uncertainty, the Bayesian approach leads one towards the presumption of a fat tail

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[22] For instance, *ceteris paribus* the new features of the standard PAGE09 model alone result in at least a threefold increase of the SCCO$_2$ estimates with respect to the earlier PAGE 2002 model.

(in the absence of reliable information to the contrary),[23] which would suggest a bias towards the Pareto tails in this research. However, whenever damages are bounded (as they are in IAMs), the weight given to different events (especially extreme negative events) is of principal concern, more than the mathematical functional forms themselves.[24]

The potential for a change in the tail probabilities to cause up to an approximate doubling of the mean value of the $SCCO_2$ and sevenfold increase in the 99th percentile is an important result from our analysis. As constructed here, the effect of adjusting the climate sensitivity parameter exceeds that of adjusting the damage function. Not only do both impact on the $SCCO_2$ mean values, but more especially on its higher percentiles. It is worth noting the high values for the $SCCO_2$ emerging from a minority of runs. For example, our results suggest that it is highly unlikely that the damage done by emitting a single tonne of CO_2 is in excess of a thousand dollars, but the possibility is not vanishingly small. Indeed, if the Pareto tails are accepted for both climate sensitivity and the damage exponents, the probability that the $SCCO_2$ exceeds US$1,000 is 4%.

The limitations of IAMs should be taken into account when interpreting the results. Specifically in relation to the dismal theorem Weitzman 2009b, the methodology developed in this paper goes some way towards incorporating uncertainty into some key elements of the model, but does not attempt to test the limit of this particular critique.[25] Furthermore, the impact of the discount rate has not been investigated, and PAGE09 standard values have been used. It is clear from the literature, and from experimentation with the model, that raising (lowering) the pure time preference rate would lower (raise) the $SCCO_2$.

The relationship of the $SCCO_2$ to Pigouvian taxation makes it an important figure for policy makers. The weight placed on extreme outcomes for policy purposes depends on the level of risk aversion. In fact, as there is uncertainty

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[23] The position is explained in Weitzman (2009b).

[24] It would, of course, have been possible to introduce a thin-tailed distributions that would have caused higher values for the $SCCO_2$, by adjusting the means and standard deviations (though the justification for doing so would have been somewhat arbitrary). This argument is expanded upon in Pindyck (2011).

[25] Weitzman (2011) emphasizes that the key "fat tail" is that of the PDF of the log of overall disutility of climate change, resulting from a chain of uncertain, interacting components.

about the PDF of many parameters, the concept of ambiguity aversion (beyond risk aversion) is also applicable in this context. A higher risk/ambiguity aversion gives more consideration to the negative extremes, which leads to the notion of climate policy being justified, in part, as insurance against catastrophe.

Disclaimer

The views expressed are purely those of the authors and may not in any circumstances be regarded as stating an official position of the European Commission.

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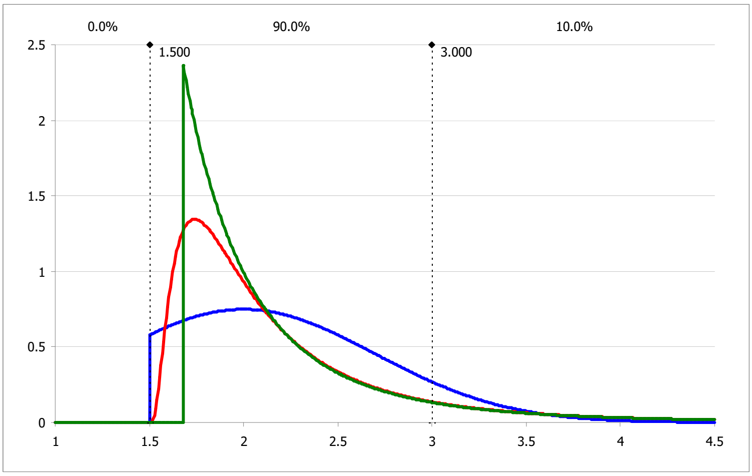

Figure 3: The full normal, lognormal and Pareto PDFs used for the economic damage exponent

A Robustness Check on PDFs: Comparisons with full normal / lognormal / Pareto PDFs

A robustness check to verify that the upper 50 percentiles of the distributions used for the climate sensitivity and damage exponent parameters are being correctly implemented is provided here. Additional runs using full normal, lognormal and Pareto PDFs in place of the combined PDFs in the main text are presented.

Figure 3 shows the full distributions for the normal, lognormal and Pareto, the upper 50 percentiles of which were used for the economic damage exponent in the main text (Figure 2).

As can be seen, using the same distributions for the lower 50 percentiles would lead to large variations in this part of the distribution. This is especially the case for the normal (blue) and Pareto (green) distributions. The normal distribution is symmetrical and so would include small and negative[26] values. The Pareto has a minimum of 1.68 and has the odd shape where the lowest value is the mode.

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[26] If used, negative values would imply that damage is very high for very low temperature rises and reduces as the temperature increases.

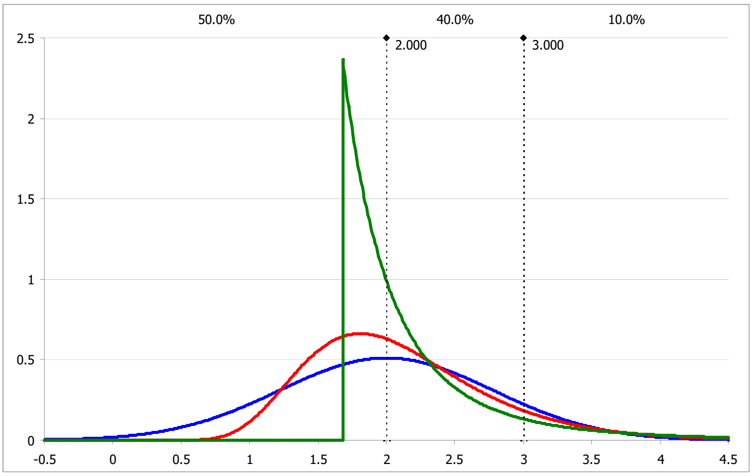

Figure 4: The adjusted full normal, lognormal and Pareto PDFs used for the economic damage exponent robustness checks

Adjustments can be made so as to make the full distributions more reflective of the original parameters. In Figure 4, such adjustments are shown for the economic damage exponent: the minimum values for the normal and lognormal are set to the minimum PAGE09 value of 1.5. Specifically, the normal is truncated at 1.5, keeping the mode at 2; the distribution is adjusted such that there is again only 10% probability above the PAGE09 maximum of 3. The location of the lognormal is increased from zero to 1.5, with the other two reference points–a median of 2 and a 90thpercentile of 3–being maintained. The Pareto distribution is unchanged from the previous figure.

For the scenarios chosen for the main text, with these full normal, lognormal and Pareto distributions large differences would be introduced into the lower half of the PDFs. There is no theoretical reason for imagining that changes in the tail shape (which are the focus of the investigation) would also be related to such change in the lower half of the distribution. In fact, this part of the distribution is expected to be the best specified of all. Nevertheless, running these distributions is carried out as a robustness check.

The adjustments to the economic and non-economic damage exponents were done as in Figure 4. For the climate sensitivity and the sea-level damage exponents, the same rules were applied. Namely, for the full Pareto version, the whole of the Pareto PDFs described in the main text were used instead of replacing the lower 50 percentiles with the PAGE09 standard values. No further changes were made. For the full lognormal, the location was set at 1.1665[27] for the climate sensitivity parameter and at 0.5 for the sea-level damage exponent. The 50th and 90th percentiles remained at the same values as before. For the full normal, the distributions were truncated at these same minimum values.[28] The mode and the 90thpercentiles were both maintained.

The results for the full normal, lognormal and Pareto distributions are shown in Table 4. The comparable results using the combined distributions from Table 3 are included for comparison.

Table 4: Alternative SENS & damage exponents with full normal, lognormal and Pareto PDFs - $SCCO_2$ in US$/$tCO_2$

	Stand. (tr.)	Thin tail (norm.)	Full norm.	Interm tail (logn.)	Full logn.	Fat tail (Par.)	Full Pareto
	Disc.ON	Disc.OFF					
Mean	**102**	**135**	**154**	**147**	**194**	**218**	**235**
5thp.	11	12	6	12	10	11	16
50th p.	49	58	66	57	57	55	65
95th p.	231	489	577	551	785	839	925
99th p.	447	1,276	1,433	1,660	2,635	3,082	3,410

Source: Authors' calculations, each based on 10,000 runs of the modified PAGE09.

The alternative methodology employed here gives somewhat different results for the $SCCO_2$ (all the three considered distributions have higher mean values), all of which can be readily explained from observation of the functional forms.

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[27] This is the theoretical minimum value for the climate sensitivity parameter in the PAGE09 standard model.

[28] i.e. 1.1665 for the climate sensitivity parameter and 0.5 for the sea-level damage exponent.

The full normal distribution has a mean of 154 $/t$CO_2$, up from 135 $/t$CO_2$. This is due to the need to truncate the distribution at a reasonable value. Choosing to truncate at the PAGE09 minimum values (e.g. 1.5 for the economic damage exponent), the extra probability mass that otherwise would be in the lower tail must be redistributed. The tail above the PAGE09 maximum value (e.g. 3 for the economic damage exponent) is adjusted to hold only 10% of the probability. This results in more probability of very low values, hence the 5th percentile of 6. However, this effect is dominated by the extra probability mass between the former median value and the PAGE09 maximum value (e.g. between 2 and 3 for the economic damage exponent). In fact, the median value is shifted upward (e.g. to 2.21 for the economic damage exponent).

The full lognormal distribution has a mean of 194 $/t$CO_2$, up from 147 $/t$CO_2$. The cause of this is the need to shift the minimum value of the distribution (from 0 to 1.5), which alters the shape of the upper 50 percentiles. Low and middle values are similar, hence the slightly lower 5th percentile and near-identical median value. The main effect is on the shape of the tail. The adjustment to the minimum value (but not to the median or 85th/90th percentiles) causes a steeper downward slope of the curve between the median and PAGE09 maximum value, which in turn causes the distribution to have a much fatter tail. In fact, the tail nearly matches the Pareto tail (e.g. for the economic damage exponent the 99thpercentile for the lognormal is 5.17, whereas for the Pareto it is only slightly higher at 5.36).

The full Pareto distribution has a mean of 235 $/t$CO_2$, up from 218 $/t$CO_2$. As the upper 50 percentiles are exactly the same as above, the changes are entirely caused by changes in the lower 50 percentiles. The minimum value of the Pareto is higher than the minimum for the standard PAGE09 first 50 percentiles (e.g. for the economic damage exponent it is 1.68 instead of 1.5). On average, this raises the values for this section of the distribution, which especially raises all the percentile values shown, including the 5thand 50th.

In the light of the robustness check provided, the methodology proposed in the main text and the functional forms used there are considered to be preferable.

B Full Results Table

Table 5 details all the results for the $SCCO_2$, each based on 10,000 runs. Most of these results appear in Tables 1, 2 and 3, which are also accompanied by more detailed explanations. Here the results are placed all together to allow direct comparisons and also to report three "intermediate" results for SENS with a normal, lognormal and Pareto tail with the discontinuity damages switched off.

Table 5: Full Results Table – $SCCO_2$ in US$/$tCO_2$ each based on 10,000 runs of PAGE09

			DAMAGE EXPONENTS				
			Standard	Standard	Normal	Lognorm	Pareto
			Disc. ON	Discontinuity damages OFF			
SENS	Standard	**Mean**	**102**	**76**	**99**	**94**	**114**
		5th perc.	11	11	11	11	11
		50th perc.	49	48	50	50	50
		95th perc.	231	226	300	300	358
		99th perc.	447	418	658	762	1,421
	Normal	**Mean**	**131**	**107**	**135**		
		5th perc.	12	12	12		
		50th perc.	57	57	58		
		95th perc.	374	374	489		
		99th perc.	841	744	1,276		
	Lognormal	**Mean**	**146**	**120**		**147**	
		5th perc.	11	11		12	
		50th perc.	57	56		57	
		95th perc.	409	412		551	
		99th perc.	1,095	1,045		1,660	
	Pareto	**Mean**	**188**	**162**			**218**
		5th perc.	12	12			11
		50th perc.	54	53			55
		95th perc.	564	549			839
		99th perc.	2,797	1,996			3,082

Source: Authors' calculations, each based on 10,000 runs of the modified PAGE09.